pends on a high temperature equilibrium that can be studied and predicted based on earth igneous rock.

In considering space manufacturing, an important criterion should be the payout time on a weight basis. This is the time required to produce a weight of product equal to the weight of plant brought from earth. The weight of plant necessary to produce 5 lb/hr of oxygen, including a nuclear power unit, is estimated to be in the order of 5 to 7 tons. Thus, such a plant would pay for itself on a weight basis in three or four months.

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A Nonlinear Lifting-Surface Theory Especially for Low-Aspect-Ratio Wings

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This note briefly summarizes a recently published[1] lifting-surface theory of wings with arbitrary planform in incompressible flow including nonlinear effects. Wings of small aspect ratio having sharp leading edges, as is well known, give rise to nonlinear dependence of lift and pitching moment on the angle of attack. This nonlinearity is due to the characteristic feature of the flow past a low-aspect-ratio wing, namely the strong cross flow causing the flow to separate at the leading or side edges and thus leading to the formation of free vortices on the upper surface of the wing.

All existing lifting-surface theories[2–4] are linear theories, which means that they can predict only a linear dependence of lift and pitching moment on angle of attack, because a given wing is replaced by a certain system of bound and free vortices, all of which are distributed in one plane.

The simple vortex model of the classical linear wing theory, namely one single vortex sheet, was modified in such a way that, by taking into account the special feature of the flow on low-aspect-ratio wings, a nonlinear dependence of lift and pitching moment on angle of attack could be obtained. The new vortex model used in this nonlinear theory is explained in Fig. 1. As in the linear theory of Truckenbrodt,[2] the wing is substituted by a number of elementary wings of infinitesimal span dy (Fig. 1a). Each of these elementary wings has a chordwise vortex distribution. Consequently, from each point over the chord length free vortices flow off downstream which, according to the vortex model employed, form an angle of $\alpha/2$ with the wing plane (Fig. 1b). A vortex model of this kind first was used by Bollay,[5] but for an entire single rectangular wing instead of the elementary

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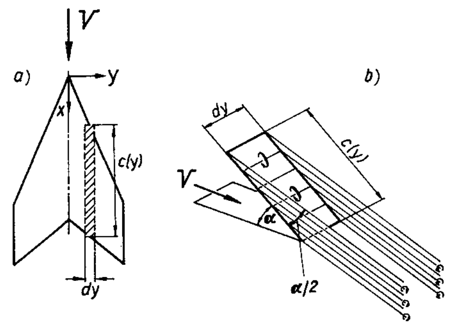

Fig. 1 Vortex model for nonlinear lifting-surface theory

wings. By this way the flowing off of free vortices at the rear at an angle of $\alpha/2$ is achieved at any point on the wing where vortex intensity is changing. In the limiting case of very small angles of attack ($\alpha \to 0$), the vortex model becomes the usual model of the linear theory.

The problem is now to determine the induced velocities at the wing area in the vortex model thus defined and to establish the flow conditions at the wing surface which then yield the equation for the unknown vortex distribution. If $k(x,y)$ is the continuous distribution of vorticity over the wing surface, the outlined vortex configuration gives the following equation for the induced angle of incidence on the wing surface:

$$\alpha_{\text{loc}} = L_1(k) + (\alpha/|\alpha|)\alpha\, L_2(k) + \ldots \tag{1}$$

where terms in powers of α of higher than first order are neglected. The linear operators $L_1(k)$ and $L_2(k)$ are determined by

$$L_1(k) = \frac{1}{4\pi V}\iint_S \frac{1}{y-y'}\frac{\partial}{\partial y'}\left[k(x',y') \times \left(1-\frac{x-x'}{R}\right)\right]dx'dy' \tag{2}$$

where $R = [(x-x')^2 + (y-y')^2]^{1/2}$ and

$$L_2(k) = -\frac{1}{8V}\frac{\partial}{\partial y}\int_{x_l}^{x_t}\left(1+\frac{x-x'}{|x-x'|}\right) \times (x-x')\,k(x',y)dx' \tag{3}$$

L_1 and L_2 have the following properties:

$$\begin{aligned} L(ck) &= cL(k) \\ L(k_1+k_2) &= L(k_1) + L(k_2) \end{aligned} \tag{4}$$

x_l, x_t are the abscissas of leading edge and trailing edge, respectively. The solution of Eq. (1) can be given in the form

$$k = k_1\alpha + k_2(\alpha/|\alpha|)\alpha^2 \tag{5}$$

Using the relations in Eq. (4), the following two equations serve for the determination of the functions k_1 and k_2:

$$L_1(k_1) = \alpha_{\text{loc}}/\alpha \tag{6}$$

$$L_1(k_2) = -(\alpha_{\text{loc}}/\alpha)L_2(k_1) \tag{7}$$

Equation (6) is identical with the integral equation of the linear theory, which can be solved by one of the well-known methods, e.g., Ref. 2. The nonlinear term is obtained from Eq. (7), which differs from Eq. (6) only by the right-hand side. Having determined k_1 from Eq. (6), it is easy to establish the right-hand side of Eq. (7), which can be solved by the same methods used in the linear theories. After functions k_1 and k_2 have been determined, the aerodynamic coefficients c_L and c_M can be obtained from the vorticity distribution $k(x,y)$ in the usual way.

By use of this method, the lift distributions and aerodynamic coefficients for series of rectangular wings, swept

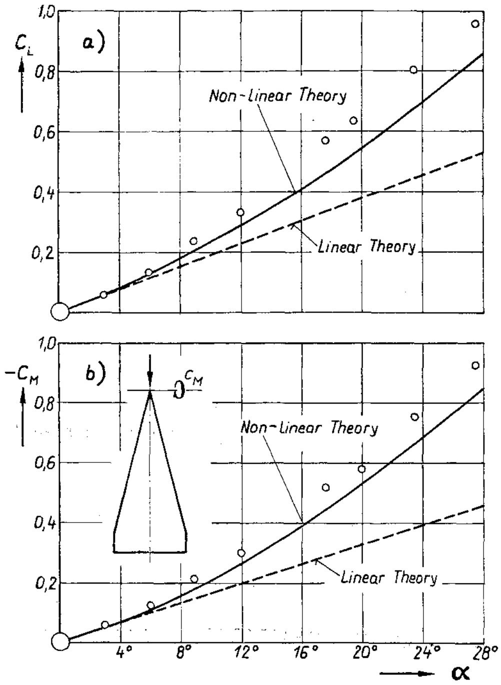

Fig. 2 Lift coefficient a) and pitching-moment coefficient b) for a slender delta wing (aspect ratio AR = 0.78, taper ratio λ = 0.125) vs angle of attack

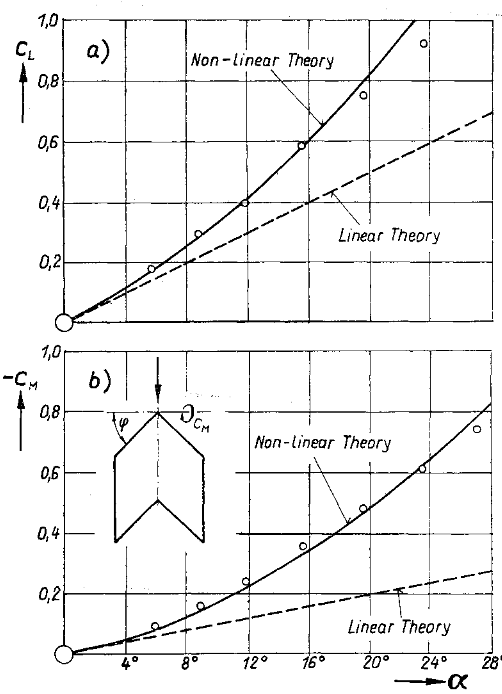

Fig. 3 Lift coefficient a) and pitching-moment coefficient b) for a swept wing (aspect ratio AR = 1, sweep angle φ = 45°) vs angle of attack

wings, and delta wings were calculated and, as far as possible, compared with measurements. The agreement is quite satisfactory. In Figs. 2 and 3, comparisons between theory and experiments are shown for a slender delta wing and for a swept wing of 45° sweep angle and taper ratio 1. For the lift coefficients as well as for the pitching-moment coefficients, the agreement is very good up to high values of angle of attack. For the drag coefficient of wings having sharp leading edges, one obtains $c_D = c_L\alpha$, since suction forces are zero due to leading-edge separation.

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Cylindrical Heat Flow with Arbitrary Heating Rates

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In this note, Chen's solution[1] is extended to the problem of purely radial heat flow through a hollow cylinder ($a \leqslant r \leqslant b$) under an arbitrary time-dependent heat flux at the outer surface ($r = b$) and zero heat flux at the internal boundary ($r = a$). The solution should be useful in current aerospace problems for stations of a missile body not influenced by nose tapering. The missile's skin material is assumed to have physical properties independent of temperature, so that the temperature $T(r,t)$ is a function of radius r and time t only.

The basic differential equation and boundary conditions can be written in the form

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} \qquad (1)$$

with

$$T(r,0) = 0 \qquad (2)$$

for $t = 0$, $a \leqslant r \leqslant b$, and

$$k\frac{\partial T(b,t)}{\partial r} = Q(t) \qquad k\frac{\partial T(a,t)}{\partial t} = 0 \qquad (3)$$

where $Q(t)$ is the heat flux at the external boundary. Using the Laplace transform,

$$\bar{T}(r,p) = \int_0^\infty e^{-pt}T(r,t)dt = \mathcal{L}\{T(r,t)\} \qquad (4)$$

these equations become

$$q^2\bar{T} = \frac{\partial^2\bar{T}}{\partial r^2} + \frac{1}{r}\frac{\partial\bar{T}}{\partial r} \qquad (\alpha q^2 = p) \qquad (5)$$

with

$$k[\partial\bar{T}(b,p)/\partial r] = \bar{Q}(p) \qquad (6)$$

and

$$k[\partial\bar{T}(a,p)/\partial r] = 0 \qquad (7)$$

The operator form of the solution is

$$\bar{T}(r,p) = \frac{\bar{Q}(p)[I_0(qr)K_1(qa) + K_0(qr)I_1(qa)]}{kq[I_1(qb)K_1(qa) - I_1(qa)K_1(qb)]} \qquad (8)$$

where I_0, I_1, K_0, K_1 are modified Bessel functions of the first

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